

ANALYSIS OF THE VERTEXES  $\Omega_Q^* \Omega_Q \phi$  AND RADIATIVE DECAYS  $\Omega_Q^* \rightarrow \Omega_Q \gamma$ Zhi-Gang Wang<sup>1</sup>

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**Abstract**

In this article, we study the vertexes  $\Omega_Q^* \Omega_Q \phi$  with the light-cone QCD sum rules, then assume the vector meson dominance of the intermediate  $\phi(1020)$ , and calculate the radiative decays  $\Omega_Q^* \rightarrow \Omega_Q \gamma$ .

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**Key Words:** Heavy baryons; Light-cone QCD sum rules**1 Introduction**

In 2006, the BaBar collaboration reported the first observation of the  $\frac{3}{2}^+$  baryon  $\Omega_c^*$  in the radiative decay  $\Omega_c^* \rightarrow \Omega_c \gamma$ , where the  $\frac{1}{2}^+$  baryon  $\Omega_c$  was reconstructed in decays to the final states  $\Omega^- \pi^+$ ,  $\Omega^- \pi^+ \pi^0$ ,  $\Omega^- \pi^+ \pi^- \pi^+$  and  $\Xi^- K^- \pi^+ \pi^+$  [1]. The  $\Omega_c^*$  lies about  $70.8 \pm 1.0 \pm 1.1 \text{ MeV}$  above the  $\Omega_c$ , and is the last singly-charm baryon with zero orbital momentum observed experimentally [2]. In 2008, the D0 collaboration reported the first observation of the doubly strange baryon  $\Omega_b^-$  in the decay channel  $\Omega_b^- \rightarrow J/\psi \Omega^-$  (with  $J/\psi \rightarrow \mu^+ \mu^-$  and  $\Omega^- \rightarrow \Lambda K^- \rightarrow p \pi^- K^-$ ) in  $p\bar{p}$  collisions at  $\sqrt{s} = 1.96 \text{ TeV}$  [3]. The experimental value  $M_{\Omega_b^-} = 6.165 \pm 0.010 \pm 0.013 \text{ GeV}$  is about  $0.1 \text{ GeV}$  larger than the existing theoretical calculations (see Ref. [4] for a short review on the relevant literatures); however, the CDF collaboration did not confirm the measured mass [5], i.e. they observed the mass of the  $\Omega_b^-$  is about  $6.0544 \pm 0.0068 \pm 0.0009 \text{ GeV}$ , which is consistent with the existing theoretical calculations.

By now, the  $\frac{1}{2}^+$  antitriplet states ( $\Lambda_c^+$ ,  $\Xi_c^+$ ,  $\Xi_c^0$ ), and the  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  sextet states ( $\Omega_c$ ,  $\Sigma_c$ ,  $\Xi_c'$ ) and ( $\Omega_c^*$ ,  $\Sigma_c^*$ ,  $\Xi_c'^*$ ) have been well established; while the corresponding  $S$ -wave bottom baryons are far from complete, only the  $\Lambda_b$ ,  $\Sigma_b$ ,  $\Sigma_b^*$ ,  $\Xi_b$ ,  $\Omega_b$  have been observed [6]. Those heavy baryons are particularly interesting for studying dynamics of the light quarks in the presence of a heavy quark. In the heavy quark limit, the three light quarks form an  $SU(3)$  flavor triplet,  $\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$ , two light quarks can form diquarks of a symmetric sextet and an antisymmetric antitriplet [7, 8].

The light-cone QCD sum rules are a powerful theoretical tool in studying the ground state heavy baryons, they carry out the operator product expansion near the light-cone  $x^2 \approx 0$  instead of the short distance  $x \approx 0$ , while the nonperturbative hadronic matrix elements are parameterized by the light-cone distribution amplitudes instead of the vacuum condensates [9, 10, 11]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal.

In Ref.[12], we assume the charm mesons  $D_{s0}(2317)$  and  $D_{s1}(2460)$  with the spin-parity  $0^+$  and  $1^+$  respectively are the conventional  $c\bar{s}$  states, and calculate the strong coupling constants  $\langle D_s^* \phi | D_{s0} \rangle$  and  $\langle D_s \phi | D_{s1} \rangle$  with the light-cone QCD sum rules, then

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take the vector meson dominance of the intermediate  $\phi(1020)$ , study the radiative decays  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$ . In previous works [4, 13] (see also Refs.[14, 15]), we have calculated the masses and the pole residues of the  $\frac{1}{2}^+$  heavy baryons  $\Omega_Q$  and the  $\frac{3}{2}^+$  heavy baryons  $\Omega_Q^*$  with the QCD sum rules. In this article, we extend our previous works to study the vertexes  $\Omega_Q^* \Omega_Q \phi$  with the light-cone QCD sum rules<sup>2</sup>, then assume the vector meson dominance of the intermediate  $\phi(1020)$ , and calculate the radiative decays  $\Omega_Q^* \rightarrow \Omega_Q \gamma$ . In Ref.[16], Aliev et al study the radiative decays  $\Sigma_Q^* \rightarrow \Sigma_Q \gamma$ ,  $\Xi_Q^* \rightarrow \Xi_Q \gamma$  and  $\Sigma_Q^* \rightarrow \Lambda_Q \gamma$  with the light cone QCD sum rules.

There have been many works dealing with the strong coupling constants of the pseudoscalar (scalar) octet mesons and vector nonet mesons with the baryons. The  $\rho NN$ ,  $\rho \Sigma \Sigma$ ,  $\rho \Xi \Xi$  and other strong coupling constants of the nonet vector mesons with the octet baryons have been calculated using the light cone QCD sum rules [17, 18, 19]. In Refs.[20, 21], Aliev et al study the strong coupling constants of the pseudoscalar octet mesons with the octet (and decuplet) baryons comprehensively. In Refs.[22, 23], we study the strong decays  $\Delta^{++} \rightarrow p \pi$ ,  $\Sigma^* \rightarrow \Sigma \pi$  and  $\Sigma^* \rightarrow \Lambda \pi$  using the light-cone QCD sum rules. Moreover, the coupling constants of the vector mesons  $\rho$  and  $\omega$  with the baryons are studied with the external field QCD sum rules [24].

The article is arranged as: in Section 2, we derive the strong coupling constants  $g_1$ ,  $g_2$  and  $g_3$  of the vertexes  $\Omega_Q^* \Omega_Q \phi$  with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.

## 2 The vertexes $\Omega_Q^* \Omega_Q \phi$ with light-cone QCD sum rules

We parameterize the vertexes  $\Omega_Q^* \Omega_Q \phi$  with three tensor structures due to Lorentz invariance and introduce three strong coupling constants  $g_1$ ,  $g_2$  and  $g_3$  [25],

$$\begin{aligned} \langle \Omega_Q(p+q) | \Omega_Q^*(p) \phi(q) \rangle &= \bar{U}(p+q) [g_1(q_\mu \epsilon_\mu - \epsilon_\mu q) \gamma_5 + g_2(P \cdot \epsilon q_\mu - P \cdot q \epsilon_\mu) \gamma_5 \\ &\quad + g_3(q \cdot \epsilon q_\mu - q^2 \epsilon_\mu) \gamma_5] U^\mu(p) \\ &= \epsilon_\mu \bar{U}(p+q) \Gamma^{\mu\nu} U_\nu(p), \end{aligned} \quad (1)$$

where the  $U(p)$  and  $U_\mu(p)$  are the Dirac spinors of the heavy baryons  $\Omega_Q$  and  $\Omega_Q^*$  respectively, the  $\epsilon_\mu$  is the polarization vector of the meson  $\phi(1020)$ , and  $P = \frac{2p+q}{2}$ .

In the following, we write down the two-point correlation function  $\Pi_\mu(p, q)$ ,

$$\Pi_\mu(p, q) = i \int d^4x e^{-ip \cdot x} \langle 0 | T \{ J(0) \bar{J}_\mu(x) \} | \phi(q) \rangle, \quad (2)$$

$$\begin{aligned} J(x) &= \epsilon^{ijk} s_i^T(x) C \gamma_\mu s_j(x) \gamma_5 \gamma^\mu Q_k(x), \\ J_\mu(x) &= \epsilon^{ijk} s_i^T(x) C \gamma_\mu s_j(x) Q_k(x), \end{aligned} \quad (3)$$

where  $Q = c$  and  $b$ , the  $i, j, k$  are color indexes, the Ioffe type heavy baryon currents  $J(x)$  and  $J_\mu(x)$  interpolate the  $\frac{1}{2}^+$  baryons  $\Omega_Q$  and the  $\frac{3}{2}^+$  baryons  $\Omega_Q^*$  respectively [4, 13], the external vector state  $\phi(1020)$  has the four momentum  $q_\mu$  with  $q^2 = M_\phi^2$ .

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<sup>2</sup>The results of the strong coupling constants among the nonet vector mesons, the octet baryons and the decuplet baryons will be presented elsewhere.

Basing on the quark-hadron duality [26, 27], we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators  $J(x)$  and  $J_\mu(x)$  into the correlation function  $\Pi_\mu(p, q)$  to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the baryons  $\Omega_Q$  and  $\Omega_Q^*$ , we get the following result,

$$\begin{aligned}
\Pi_\mu(p, q) &= \frac{\langle 0|J(0)|\Omega_Q(q+p)\rangle\langle\Omega_Q(q+p)|\Omega_Q^*(p)\phi(q)\rangle\langle\Omega_Q^*(p)|\bar{J}_\mu(0)|0\rangle}{\left[M_{\Omega_Q}^2 - (q+p)^2\right]\left[M_{\Omega_Q^*}^2 - p^2\right]} + \dots \\
&= \frac{\lambda_{\Omega_Q}\lambda_{\Omega_Q^*}}{\left[M_{\Omega_Q}^2 - (q+p)^2\right]\left[M_{\Omega_Q^*}^2 - p^2\right]} \left\{ g_1 \left[ M_{\Omega_Q} + M_{\Omega_Q^*} \right] \not{q} \not{p} \gamma_5 q_\mu \right. \\
&\quad - g_1 \left[ M_{\Omega_Q} + M_{\Omega_Q^*} \right] \not{q} \not{p} \gamma_5 \epsilon_\mu + g_2 \not{q} \not{p} \gamma_5 p \cdot \epsilon q_\mu - g_2 \not{q} \not{p} \gamma_5 q \cdot p \epsilon_\mu \\
&\quad \left. - \frac{g_2}{2} \not{q} \not{p} \gamma_5 q^2 \epsilon_\mu - g_3 \not{q} \not{p} \gamma_5 q^2 \epsilon_\mu + \dots \right\} + \dots, \tag{4}
\end{aligned}$$

where the following definitions have been used,

$$\begin{aligned}
\langle 0|J(0)|\Omega_Q(p)\rangle &= \lambda_{\Omega_Q} U(p, s), \\
\langle 0|J_\mu(0)|\Omega_Q^*(p)\rangle &= \lambda_{\Omega_Q^*} U_\mu(p, s), \\
\sum_s U(p, s) \bar{U}(p, s) &= \not{p} + M_{\Omega_Q}, \\
\sum_s U_\mu(p, s) \bar{U}_\nu(p, s) &= -(\not{p} + M_{\Omega_Q^*}) \left( g_{\mu\nu} - \frac{\gamma_\mu \gamma_\nu}{3} - \frac{2p_\mu p_\nu}{3M_{\Omega_Q^*}^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3M_{\Omega_Q^*}} \right). \tag{5}
\end{aligned}$$

The current  $J_\mu(x)$  couples not only to the spin-parity  $J^P = \frac{3}{2}^+$  states, but also to the spin-parity  $J^P = \frac{1}{2}^-$  states. For a generic  $\frac{1}{2}^-$  resonance  $\tilde{\Omega}_Q^*$ ,

$$\langle 0|J_\mu(0)|\tilde{\Omega}_Q^*(p)\rangle = \lambda_*(\gamma_\mu - 4\frac{p_\mu}{M_*})U^*(p, s), \tag{6}$$

where  $\lambda^*$  is the pole residue and  $M_*$  is the mass. The spinor  $U^*(p, s)$  satisfies the usual Dirac equation  $(\not{p} - M_*)U^*(p) = 0$ . If we choose the tensor structures  $\not{q} \not{p} \gamma_5 q_\mu$ ,  $\not{q} \not{p} \gamma_5 p \cdot \epsilon q_\mu$ ,  $\not{q} \not{p} \gamma_5 \epsilon_\mu$ , the baryon  $\tilde{\Omega}_Q^*$  has no contamination.

In the following, we briefly outline the operator product expansion for the correlation function  $\Pi_\mu(p, q)$  in perturbative QCD theory. The calculations are performed at the large space-like momentum regions  $(q+p)^2 \ll 0$  and  $p^2 \ll 0$ , which correspond to the small light-cone distance  $x^2 \approx 0$  required by the validity of the operator product expansion approach. We write down the "full" propagator of a massive quark in the presence of the

quark and gluon condensates firstly [9, 27],

$$\begin{aligned}
S_{ij}(x) &= \frac{i\delta_{ij}\not{x}}{2\pi^2x^4} - \frac{\delta_{ij}m_s}{4\pi^2x^2} - \frac{\delta_{ij}}{12}\langle\bar{s}s\rangle + \frac{i\delta_{ij}}{48}m_s\langle\bar{s}s\rangle\not{x} - \frac{\delta_{ij}x^2}{192}\langle\bar{s}g_s\sigma Gs\rangle \\
&\quad + \frac{i\delta_{ij}x^2}{1152}m_s\langle\bar{s}g_s\sigma Gs\rangle\not{x} - \frac{i}{16\pi^2x^2}\int_0^1 dv G_{\mu\nu}^{ij}(vx)[(1-v)\not{x}\sigma^{\mu\nu} + v\sigma^{\mu\nu}\not{x}] + \dots, \\
S_Q^{ij}(x) &= \frac{i}{(2\pi)^4}\int d^4k e^{-ik\cdot x}\left\{\frac{\delta_{ij}}{\not{k}-m_Q} - \frac{g_s G_{ij}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_Q) + (\not{k}+m_Q)\sigma_{\alpha\beta}}{(k^2-m_Q^2)^2}\right. \\
&\quad \left. + \frac{\pi^2}{3}\langle\frac{\alpha_s GG}{\pi}\rangle\delta_{ij}m_Q\frac{k^2+m_Q}{(k^2-m_Q^2)^4}\not{k} + \dots\right\}, \tag{7}
\end{aligned}$$

where  $\langle\bar{s}g_s\sigma Gs\rangle = \langle\bar{s}g_s\sigma_{\alpha\beta}G^{\alpha\beta}s\rangle$  and  $\langle\frac{\alpha_s GG}{\pi}\rangle = \langle\frac{\alpha_s G_{\alpha\beta}G^{\alpha\beta}}{\pi}\rangle$ , then contract the quark fields in the correlation function  $\Pi_\mu(p, q)$  with Wick theorem, and obtain the result:

$$\begin{aligned}
\Pi_\mu(p, q) &= 2i\epsilon^{ijk}\epsilon^{i'j'k'}\int d^4x e^{-ip\cdot x} \\
&\quad \left\{\gamma_5\gamma^\alpha S_Q^{kk'}(-x)Tr[\gamma_\alpha\langle 0|s_j(0)\bar{s}_{j'}(x)|\phi(q)\rangle\gamma_\mu CS_{ii'}^T(-x)C]\right. \\
&\quad \left.+ \gamma_5\gamma^\alpha S_Q^{kk'}(-x)Tr[\gamma_\alpha S_{jj'}(-x)\gamma_\mu C\langle 0|s_i(0)\bar{s}_{i'}(x)|\phi(q)\rangle^T C]\right\}. \tag{8}
\end{aligned}$$

Perform the following Fierz re-ordering to extract the contributions from the two-particle and three-particle  $\phi$ -meson light-cone distribution amplitudes respectively,

$$\begin{aligned}
s_\alpha^a(0)\bar{s}_\beta^b(x) &= -\frac{1}{12}\delta_{ab}\delta_{\alpha\beta}\bar{s}(x)s(0) - \frac{1}{12}\delta_{ab}(\gamma^\mu)_{\alpha\beta}\bar{s}(x)\gamma_\mu s(0) \\
&\quad - \frac{1}{24}\delta_{ab}(\sigma^{\mu\nu})_{\alpha\beta}\bar{s}(x)\sigma_{\mu\nu}s(0) \\
&\quad + \frac{1}{12}\delta_{ab}(\gamma^\mu\gamma_5)_{\alpha\beta}\bar{s}(x)\gamma_\mu\gamma_5s(0) \\
&\quad + \frac{1}{12}\delta_{ab}(i\gamma_5)_{\alpha\beta}\bar{s}(x)i\gamma_5s(0), \tag{9} \\
s_\alpha^a(0)\bar{s}_\beta^b(x)G_{\lambda\tau}^{ba}(vx) &= -\frac{1}{4}\delta_{\alpha\beta}\bar{s}(x)G_{\lambda\tau}(vx)s(0) - \frac{1}{4}(\gamma^\mu)_{\alpha\beta}\bar{s}(x)\gamma_\mu G_{\lambda\tau}(vx)s(0) \\
&\quad - \frac{1}{8}(\sigma^{\mu\nu})_{\alpha\beta}\bar{s}(x)\sigma_{\mu\nu}G_{\lambda\tau}(vx)s(0) \\
&\quad + \frac{1}{4}(\gamma^\mu\gamma_5)_{\alpha\beta}\bar{s}(x)\gamma_\mu\gamma_5G_{\lambda\tau}(vx)s(0) \\
&\quad + \frac{1}{4}(i\gamma_5)_{\alpha\beta}\bar{s}(x)i\gamma_5G_{\lambda\tau}(vx)s(0), \tag{10}
\end{aligned}$$

and replace the hadronic matrix elements (such as the  $\langle 0|\bar{s}(x)\gamma_\mu\gamma_5s(0)|\phi(q)\rangle$ , etc.) with the corresponding  $\phi$ -meson light-cone distribution amplitudes, then substitute the full  $s$  and  $Q$  quark propagators into above correlation function and complete the integral in the coordinate space, finally integrate over the variable  $k$ , we can obtain the correlation function  $\Pi_\mu(p, q)$  at the level of quark-gluon degree of freedom. In calculation, the two-particle and three-particle  $\phi$ -meson light-cone distribution amplitudes have been used [28, 29, 30, 31], the explicit definitions are given in the appendix. The parameters in the

light-cone distribution amplitudes are scale dependent and are estimated with the QCD sum rules [30, 31]. In this article, the energy scale  $\mu$  is chosen to be  $\mu = 1$  GeV.

Taking double Borel transform with respect to the variables  $Q_1^2 = -p^2$  and  $Q_2^2 = -(p+q)^2$  respectively, then subtract the contributions from the high resonances and continuum states by introducing the threshold parameter  $s_0$  (i.e.  $M^{2n} \rightarrow \frac{1}{\Gamma[n]} \int_0^{s_0} ds s^{n-1} e^{-\frac{s}{M^2}}$ ), finally we obtain six sum rules for the strong coupling constants  $g_1$ ,  $g_2$  and  $G3 = -\left(M_{\Omega_Q} + M_{\Omega_Q}^*\right) g_1 - M_\phi^2 \left(\frac{g_2}{2} + g_3\right)$  respectively, the explicit expressions are presented in the appendix.

### 3 Numerical result and discussion

The input parameters are taken as  $M_\phi = 1.019455$  GeV,  $M_{\Omega_c} = 2.6952$  GeV,  $M_{\Omega_c^*} = 2.7659$  GeV,  $M_{\Omega_b} = 6.165$  GeV [6],  $M_{\Omega_b^*} = 6.06$  GeV,  $\lambda_{\Omega_c} = (0.075 \pm 0.01)$  GeV<sup>3</sup>,  $\lambda_{\Omega_c^*} = (0.05 \pm 0.01)$  GeV<sup>3</sup>,  $\lambda_{\Omega_b} = (0.10 \pm 0.01)$  GeV<sup>3</sup>,  $\lambda_{\Omega_b^*} = (0.06 \pm 0.01)$  GeV<sup>3</sup> [4, 13],  $f_\phi = (0.215 \pm 0.005)$  GeV,  $f_\phi^\perp = (0.186 \pm 0.009)$  GeV,  $a_1^\parallel = 0.0$ ,  $a_1^\perp = 0.0$ ,  $a_2^\parallel = 0.18 \pm 0.08$ ,  $a_2^\perp = 0.14 \pm 0.07$ ,  $\zeta_3^\parallel = 0.024 \pm 0.008$ ,  $\tilde{\lambda}_3^\parallel = 0.0$ ,  $\tilde{\omega}_3^\parallel = -0.045 \pm 0.015$ ,  $\kappa_3^\parallel = 0.0$ ,  $\omega_3^\parallel = 0.09 \pm 0.03$ ,  $\lambda_3^\parallel = 0.0$ ,  $\kappa_3^\perp = 0.0$ ,  $\omega_3^\perp = 0.20 \pm 0.08$ ,  $\lambda_3^\perp = 0.0$ ,  $\varsigma_4^\parallel = 0.00 \pm 0.02$ ,  $\tilde{\omega}_4^\parallel = -0.02 \pm 0.01$ ,  $\varsigma_4^\perp = -0.01 \pm 0.03$ ,  $\tilde{\varsigma}_4^\perp = -0.03 \pm 0.04$ ,  $\kappa_4^\parallel = 0.0$ ,  $\kappa_4^\perp = 0.0$  [30, 31],  $m_s = (140 \pm 10)$  MeV,  $m_c = (1.35 \pm 0.10)$  GeV,  $m_b = (4.7 \pm 0.1)$  GeV [6],  $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle$ ,  $\langle \bar{s}g_s \sigma G s \rangle = m_0^2 \langle \bar{s}s \rangle$ ,  $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ , and  $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{ GeV})^4$  at the energy scale  $\mu = 1$  GeV [26, 27, 32].

The threshold parameters and the Borel parameters are taken as  $s_0 = (10.5 \pm 1.0) \text{ GeV}^2$  and  $M^2 = (2.2 - 3.2) \text{ GeV}^2$  in the charm channels, and  $s_0 = (44.5 \pm 1.0) \text{ GeV}^2$  and  $M^2 = (5.0 - 6.0) \text{ GeV}^2$  in the bottom channels, which are determined by the two-point QCD sum rules to avoid possible contaminations from the high resonances and continuum states [4, 13]. In Refs.[33, 34], Melikhov et al study the ground-state form-factor in an exactly solvable harmonic-oscillator model to illustrate the exact effective continuum threshold for vacuum-to-hadron correlation function is very difficult to obtain, as the effective continuum threshold maybe depend on the Borel parameter. We show the values of the strong coupling constants  $g_1$ ,  $g_2$  and  $G3$  with variation of the threshold parameters  $s_0$  in Fig.1. From the figure we can see that in the present case the numerical results are insensitive to the threshold parameters.

The main uncertainties come from the six parameters  $\lambda_{\Omega_Q}$ ,  $\lambda_{\Omega_Q}^*$  and  $m_Q$ , the variations of those parameters can lead to relatively large changes for the numerical values, refining those parameters are of great importance. Although there are many parameters in the light-cone distributions amplitudes [30, 31], the uncertainties originate from those parameters are rather small.

Taking into account all the uncertainties of the relevant parameters, finally we obtain the numerical results of the strong coupling constants  $g_1$ ,  $g_2$  and  $G3$ , which are shown in Fig.2,

$$\begin{aligned} -g_1 &= 6.95_{-1.84}^{+2.78} \text{ GeV}^{-1}, \\ -g_2 &= 1.35_{-0.71}^{+1.14} \text{ GeV}^{-2}, \\ G3 &= 25.0_{-6.7}^{+10.1}, \end{aligned} \tag{11}$$

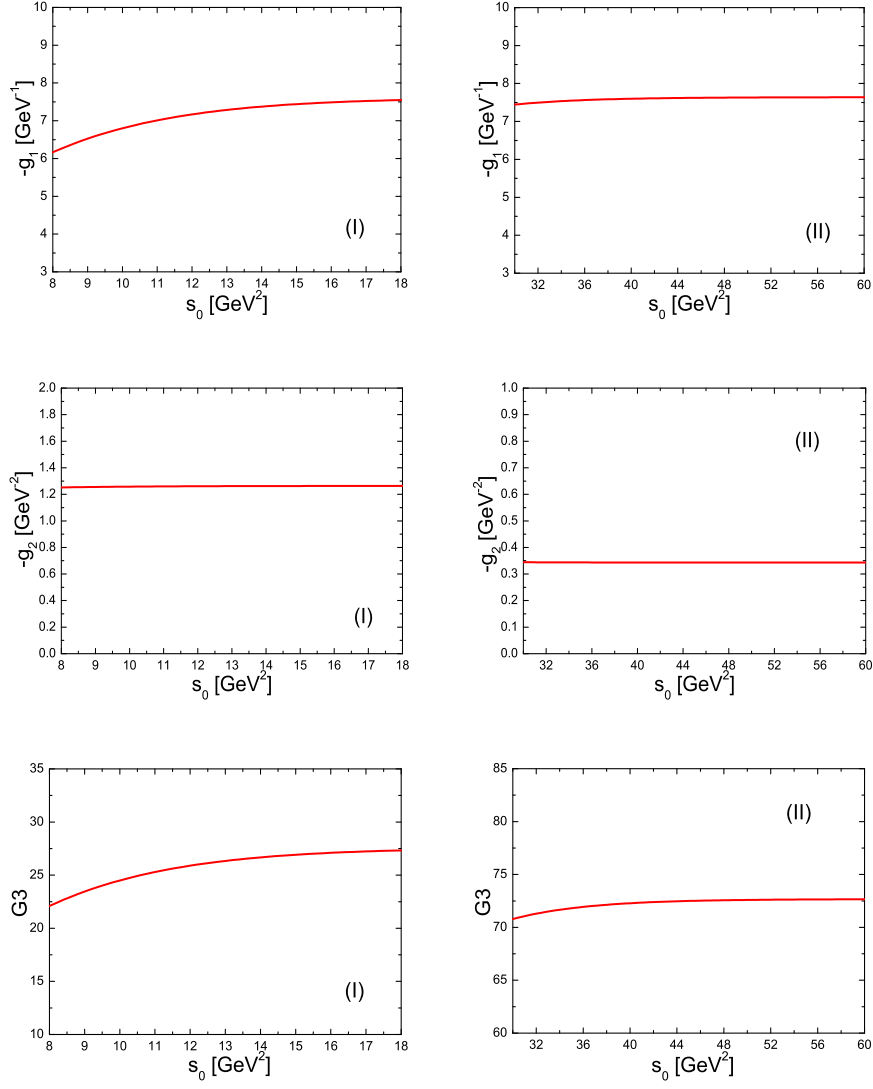


Figure 1: The strong coupling constants  $g_1$ ,  $g_2$  and  $G_3$  with variation of the threshold parameters  $s_0$ , the Borel parameters are taken to be the central values; the (I) and (II) correspond to the charm and bottom channels respectively.

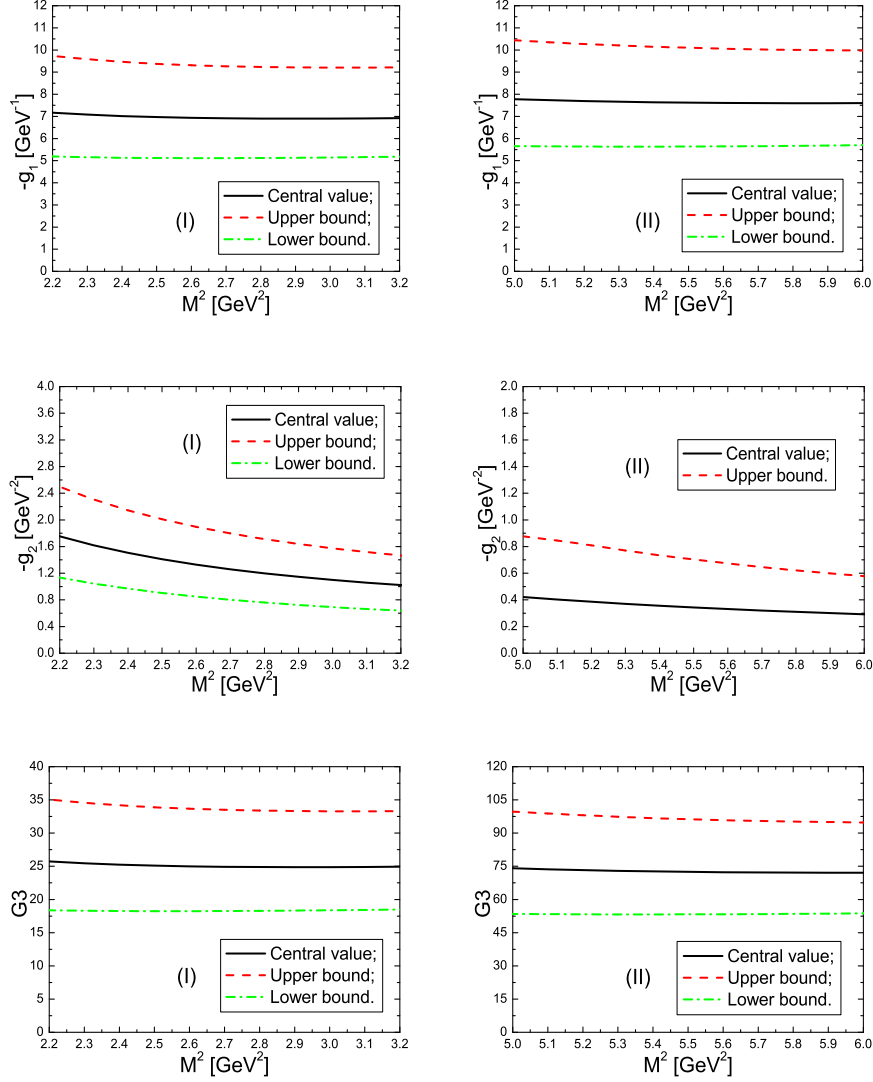


Figure 2: The strong coupling constants  $g_1$ ,  $g_2$  and  $G_3$  with variation of the Borel parameter  $M^2$ ; the (I) and (II) correspond to the charm and bottom channels respectively.

and

$$\begin{aligned}
-g_1 &= 7.63_{-2.00}^{+2.80} \text{ GeV}^{-1}, \\
-g_2 &= 0.34_{-0.34}^{+0.50} \text{ GeV}^{-2}, \\
\text{G3} &= 72.5_{-19.2}^{+27.2},
\end{aligned} \tag{12}$$

in the charm and bottom channels respectively. In this article, we calculate the uncertainties  $\delta$  with the formula

$$\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 \big|_{x_i=\bar{x}_i} (x_i - \bar{x}_i)^2}, \tag{13}$$

where the  $f$  denote strong coupling constants  $g_1$ ,  $g_2$  and  $\text{G3}$ , the  $x_i$  denote the relevant parameters  $m_Q$ ,  $\langle \bar{q}q \rangle$ ,  $\langle \bar{s}s \rangle$ ,  $\dots$ . As the partial derivatives  $\frac{\partial f}{\partial x_i}$  are difficult to carry out analytically, we take the approximation  $\left( \frac{\partial f}{\partial x_i} \right)^2 (x_i - \bar{x}_i)^2 \approx [f(\bar{x}_i \pm \Delta x_i) - f(\bar{x}_i)]^2$  in the numerical calculations.

The light-cone QCD sum rules approaches have been applied to determine the strong coupling constant  $g_{D^*D\pi}$  in the strong decay  $D^{*+} \rightarrow D^0\pi^+$  both in the leading approximation [35, 36] and the next-to-leading order approximation [37]. The discrepancy between the experimental data from the CLEO collaboration and the theoretical predictions is rather large. The upper bound  $g_{D^*D\pi} = 13.5$  ( $g_{D^*D\pi} = 10.5 \pm 3.0$  [37]) is too small to account for the experimental data,  $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$  [38, 39]. There have been several explanations, for example, Becirevic et al take into account the contribution from an explicit radial excitation to the hadronic spectral density to improve the value of  $g_{D^*D\pi}$  [40]; Kim tries to subtract the term  $M^2 e^{-\frac{s_0}{M^2}}$  which is supposed to come from a mathematically spurious term and should not be a part of the final sum rules to smear the discrepancy [41]; while Duraes et al resort to the intermediate hadronic loops to improve the predictive ability [42]. Or the simple quark-hadron duality ansatz which works in the one-variable dispersion relation might be too crude for the double dispersion relation [43]. Irrespective of the possible reasons, the light-cone sum rules cannot give satisfactory value to account for the experimental data in the channel  $D^{*+} \rightarrow D^0\pi^+$ , while the light-cone QCD sum rules are rather successful in calculating the strong coupling constants among the baryons, baryons and mesons, for example, we study the strong decays  $\Delta^{++} \rightarrow p\pi$ ,  $\Sigma^* \rightarrow \Sigma\pi$  and  $\Sigma^* \rightarrow \Lambda\pi$  using the light-cone QCD sum rules, and observe that the numerical values of the widths are in agreement with the experimental data within uncertainties [22, 23]. The present predictions for the values of the strong coupling constants  $g_1$ ,  $g_2$  and  $\text{G3}$  are reasonable.

The radiative decays  $\Omega_Q^* \rightarrow \Omega_Q \gamma$  can be described by the following electromagnetic lagrangian  $\mathcal{L}$ ,

$$\mathcal{L} = -eQ_b \bar{b} \gamma_\mu b A^\mu - eQ_c \bar{c} \gamma_\mu c A^\mu - eQ_s \bar{s} \gamma_\mu s A^\mu, \tag{14}$$

where the  $A_\mu$  is the electromagnetic field. From the lagrangian  $\mathcal{L}$ , we can obtain the decay



amplitude with the assumption of the vector meson dominance,

$$\begin{aligned}
\langle \Omega_Q(p) \gamma(q) | \mathcal{L} | \Omega_Q^*(p+q) \rangle &= -e Q_s \eta_\mu^* \langle \Omega_Q(p) | \bar{s} \gamma^\mu s | \Omega_Q^*(p+q) \rangle + \cdots \\
&= -e Q_s \eta_\mu^* f_\phi M_\phi \epsilon_\mu \frac{i}{q^2 - M_\phi^2} \langle \phi(q) \Omega_Q(p) | \Omega_Q^*(p+q) \rangle + \cdots \\
&= \frac{ie Q_s \eta_\alpha^* f_\phi}{M_\phi} U(p) \Gamma^{\alpha\beta} U_\beta(p+q) + \cdots, \tag{15}
\end{aligned}$$

where the  $\eta_\mu$  is the polarization vector of the photon. In the heavy quark limit, the matrix elements  $\langle \Omega_Q(p) | \bar{Q} \gamma_\mu Q | \Omega_Q^*(p+q) \rangle \propto M_{J/\psi(\Upsilon)}^{-\frac{3}{2}}$  and can be neglected, so we consider only the contribution of the intermediate  $\phi(1020)$ .

From the strong coupling constants  $g_1$  and  $g_2$ , we can obtain the decay widths  $\Gamma_{\Omega_Q^* \rightarrow \Omega_Q \gamma}$ ,

$$\Gamma_{\Omega_Q^* \rightarrow \Omega_Q \gamma} = \frac{\alpha (M_{\Omega_Q^*}^2 - M_{\Omega_Q}^2)}{16 M_{\Omega_Q^*}^3} \left( \frac{Q_s f_\phi}{M_\phi} \right)^2 \sum_{ss'} | \eta_\mu^* \bar{U}(p, s) \Gamma^{\mu\nu} U_\nu(p+q, s') |^2, \tag{16}$$

the numerical values are

$$\begin{aligned}
\Gamma_{\Omega_c^* \rightarrow \Omega_c \gamma} &= 1.16_{-0.54}^{+1.12} \text{ KeV}, \\
\Gamma_{\Omega_b^* \rightarrow \Omega_b \gamma} &= 0.74_{-0.34}^{+0.64} \text{ eV}. \tag{17}
\end{aligned}$$

Here we take the value  $M_{\Omega_b} = 6.0544 \text{ GeV}$  from the CDF collaboration [5], if we take the value  $M_{\Omega_b} = 6.615 \text{ GeV}$  from the D0 collaboration [3], the radiative decay  $\Omega_b^* \rightarrow \Omega_b \gamma$  is kinematically forbidden. Comparing with the values from the constituent quark model  $\Gamma_{\Omega_c^* \rightarrow \Omega_c \gamma} = 3.13 \text{ KeV}$  [44], the hyper central model  $\Gamma_{\Omega_c^* \rightarrow \Omega_c \gamma} = 0.79 \text{ KeV}$  [44], and the non-relativistic potential model  $\Gamma_{\Omega_c^* \rightarrow \Omega_c \gamma} = 0.36 \text{ KeV}$  [45], the present prediction  $\Gamma_{\Omega_c^* \rightarrow \Omega_c \gamma} = 1.16_{-0.54}^{+1.12} \text{ KeV}$  is rather good, though the uncertainty is somewhat large.

## 4 Conclusion

In this article, we parameterize the vertexes  $\Omega_Q^* \Omega_Q \phi$  with three tensor structures due to Lorentz invariance, study the corresponding three strong coupling constants with the light-cone QCD sum rules, then assume the vector meson dominance of the intermediate  $\phi(1020)$  as the contributions from the  $J/\psi$  and  $\Upsilon$  are negligible in the heavy quark limit, and calculate the radiative decay widths  $\Gamma_{\Omega_Q^* \rightarrow \Omega_Q \gamma}$ . The predictions can be compared with the experimental data in the future. The strong coupling constants in the vertexes  $\Omega_Q^* \Omega_Q \phi$  are basic parameters in describing the interactions among the heavy baryon states, once reasonable values are obtained, we can use them to perform phenomenological analysis.

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## Appendix

The light-cone distribution amplitudes of the  $\phi(1020)$  meson are defined by [28, 29, 30, 31],

$$\begin{aligned}
\langle 0 | \bar{s}(x) \gamma_\mu s(0) | \phi(q) \rangle &= q_\mu \frac{\epsilon \cdot x}{q \cdot x} f_\phi M_\phi \int_0^1 du e^{-i\bar{u}q \cdot x} \left[ \phi_{\parallel}(u) + \frac{M_\phi^2 x^2}{16} A(u) \right] \\
&+ \left[ \epsilon_\mu - q_\mu \frac{\epsilon \cdot x}{q \cdot x} \right] f_\phi M_\phi \int_0^1 du e^{-i\bar{u}q \cdot x} g_{\perp}^{(v)}(u) \\
&- \frac{1}{2} x_\mu \frac{\epsilon \cdot x}{(q \cdot x)^2} f_\phi M_\phi^3 \int_0^1 du e^{-i\bar{u}q \cdot x} C(u), \\
\langle 0 | \bar{s}(x) \sigma_{\mu\nu} s(0) | \phi(q) \rangle &= [\epsilon_\mu q_\nu - \epsilon_\nu q_\mu] i f_\phi^\perp \int_0^1 du e^{-i\bar{u}q \cdot x} \left[ \phi_{\perp}(u) + \frac{M_\phi^2 x^2}{16} A_{\perp}(u) \right] \\
&+ [q_\mu x_\nu - q_\nu x_\mu] \frac{\epsilon \cdot x}{(q \cdot x)^2} i f_\phi^\perp M_\phi^2 \int_0^1 du e^{-i\bar{u}q \cdot x} B_{\perp}(u) \\
&+ \frac{\epsilon_\mu x_\nu - \epsilon_\nu x_\mu}{2q \cdot x} i f_\phi^\perp M_\phi^2 \int_0^1 du e^{-i\bar{u}q \cdot x} C_{\perp}(u), \\
\langle 0 | \bar{s}(x) \gamma_\mu \gamma_5 s(0) | \phi(q) \rangle &= -\frac{1}{4} \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu q^\alpha x^\beta \tilde{f}_\phi M_\phi \int_0^1 du e^{-i\bar{u}q \cdot x} g_{\perp}^{(a)}(u), \\
\langle 0 | \bar{s}(x) s(0) | \phi(q) \rangle &= -\frac{i}{2} \epsilon \cdot x \tilde{f}_\phi^\perp M_\phi^2 \int_0^1 du e^{-i\bar{u}q \cdot x} h_{\parallel}^{(s)}(u), \tag{18}
\end{aligned}$$

$$\begin{aligned}
\langle 0 | \bar{s}(x) \gamma_\alpha \gamma_5 \tilde{G}_{\mu\nu}(vx) s(0) | \phi(q) \rangle &= q_\alpha [\epsilon_\mu q_\nu - \epsilon_\nu q_\mu] f_\phi M_\phi \int \mathcal{D}\alpha e^{-i(\alpha_{\bar{s}} + v\alpha_g)q \cdot x} \mathcal{A}(\alpha_i), \\
\langle 0 | \bar{s}(x) i\gamma_\alpha G_{\mu\nu}(vx) s(0) | \phi(q) \rangle &= q_\alpha [\epsilon_\mu q_\nu - \epsilon_\nu q_\mu] f_\phi M_\phi \int \mathcal{D}\alpha e^{-i(\alpha_{\bar{s}} + v\alpha_g)q \cdot x} \mathcal{V}(\alpha_i), \\
\langle 0 | \bar{s}(x) i\gamma_5 \tilde{G}_{\mu\nu}(vx) s(0) | \phi(q) \rangle &= [\epsilon_\mu q_\nu - \epsilon_\nu q_\mu] i f_\phi^\perp M_\phi^2 \int \mathcal{D}\alpha e^{-i(\alpha_{\bar{s}} + v\alpha_g)q \cdot x} \tilde{\mathcal{S}}(\alpha_i), \\
\langle 0 | \bar{s}(x) G_{\mu\nu}(vx) s(0) | \phi(q) \rangle &= [\epsilon_\mu q_\nu - \epsilon_\nu q_\mu] i f_\phi^\perp M_\phi^2 \int \mathcal{D}\alpha e^{-i(\alpha_{\bar{s}} + v\alpha_g)q \cdot x} \mathcal{S}(\alpha_i), \\
\langle 0 | \bar{s}(x) \sigma_{\alpha\beta} G_{\mu\nu}(vx) s(0) | \phi(q) \rangle &= [q_\alpha q_\mu g_{\beta\nu} - q_\beta q_\mu g_{\alpha\nu} - q_\alpha q_\nu g_{\beta\mu} + q_\beta q_\nu g_{\alpha\mu}] \\
&f_\phi^\perp M_\phi^2 \frac{\epsilon \cdot x}{2q \cdot x} \int \mathcal{D}\alpha e^{-i(\alpha_{\bar{s}} + v\alpha_g)q \cdot x} \mathcal{T}(\alpha_i), \tag{19}
\end{aligned}$$

where

$$\begin{aligned}
C(u) &= g_3(u) + \phi_{\parallel}(u) - 2g_{\perp}^{(v)} \\
B_{\perp}(u) &= h_{\parallel}^{(t)}(u) - \frac{1}{2}\phi_{\perp}(u) - \frac{1}{2}h_3(u) \\
C_{\perp}(u) &= h_3(u) - \phi_{\perp}(u) \\
\int \mathcal{D}\alpha &= \int_0^1 d\alpha_{\bar{s}} d\alpha_s d\alpha_g \delta(\alpha_{\bar{s}} + \alpha_s + \alpha_g - 1), \tag{20}
\end{aligned}$$

$\bar{u} = 1 - u$ ,  $\tilde{f}_\phi = f_\phi - f_\phi^\perp \frac{2m_s}{M_\phi}$ ,  $\tilde{f}_\phi^\perp = f_\phi^\perp - f_\phi \frac{2m_s}{M_\phi}$ , the lengthy expressions of the light-cone distribution amplitudes  $\phi_{\parallel}(u)$ ,  $\phi_{\perp}(u)$ ,  $A(u)$ ,  $A_{\perp}(u)$ ,  $g_{\perp}^{(v)}(u)$ ,  $g_{\perp}^{(a)}(u)$ ,  $h_{\parallel}^{(s)}(u)$ ,  $h_{\parallel}^{(t)}(u)$ ,  $h_3(u)$ ,

$g_3(u)$ ,  $\mathcal{A}(\alpha_i)$ ,  $\mathcal{S}(\alpha_i)$ ,  $\tilde{\mathcal{S}}(\alpha_i)$ ,  $\mathcal{T}(\alpha_i)$ ,  $\mathcal{V}(\alpha_i)$  can be found in Refs. [30, 31].

The six sum rules for the strong coupling constants  $g_1$ ,  $g_2$  and  $G_3$ ,

$$\begin{aligned}
g_1 = & \frac{1}{\lambda_{\Omega_Q} \lambda_{\Omega_Q^*} (M_{\Omega_Q} + M_{\Omega_Q^*})} \exp \frac{M_{\Omega_Q}^2 + M_{\Omega_Q^*}^2 - 2u_0(1-u_0)M_\phi^2}{2M^2} \\
& \left\{ -\frac{u_0 f_\phi M_\phi g_\perp^{(v)}(1-u_0)}{2\pi^2} M^4 E_1(x) \int_0^1 dt t(1-t) e^{-\frac{\tilde{m}_Q^2}{M^2}} \right. \\
& + \frac{u_0 m_Q^2 f_\phi M_\phi g_\perp^{(v)}(1-u_0)}{36M^2} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{m_s f_\phi^\perp \phi_\perp (1-u_0)}{2\pi^2} M^4 E_1(x) \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{m_s m_Q^2 f_\phi^\perp \phi_\perp (1-u_0)}{36M^2} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{u_0 m_s f_\phi^\perp M_\phi^2 \tilde{C}_\perp (1-u_0)}{2\pi^2} M^2 E_0(x) \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{u_0 m_s m_Q^2 f_\phi^\perp M_\phi^2 \tilde{C}_\perp (1-u_0)}{36M^4} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{u_0 \tilde{f}_\phi M_\phi}{8\pi^2} M^4 E_1(x) \frac{d}{du_0} g_\perp^{(a)}(1-u_0) \int_0^1 dt t(1-t) e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{u_0 m_Q^2 \tilde{f}_\phi M_\phi}{144M^2} \langle \frac{\alpha_s GG}{\pi} \rangle \frac{d}{du_0} g_\perp^{(a)}(1-u_0) \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{\tilde{f}_\phi M_\phi g_\perp^{(a)}(1-u_0)}{4\pi^2} M^4 E_1(x) \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{m_Q^2 \tilde{f}_\phi M_\phi g_\perp^{(a)}(1-u_0)}{72M^2} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{u_0 f_\phi M_\phi^3}{4\pi^2} M^2 E_0(x) \int_0^1 dt t \int_0^{u_0} d\alpha_{\bar{s}} \int_{u_0-\alpha_{\bar{s}}}^{1-\alpha_{\bar{s}}} d\alpha_g \frac{(1-2v)\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i)}{\alpha_g} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& \left. + \frac{f_\phi M_\phi}{4\pi^2} M^4 E_1(x) \int_0^1 dt t \frac{d}{du_0} \int_0^{u_0} d\alpha_{\bar{s}} \int_{u_0-\alpha_{\bar{s}}}^{1-\alpha_{\bar{s}}} d\alpha_g (1-v) \frac{\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i)}{\alpha_g} e^{-\frac{\tilde{m}_Q^2}{M^2}} \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\lambda_{\Omega_Q} \lambda_{\Omega_Q^*} (M_{\Omega_Q} + M_{\Omega_Q^*})} \exp \frac{M_{\Omega_Q}^2 + M_{\Omega_Q^*}^2 - 2m_Q^2 - 2u_0(1-u_0)M_\phi^2}{2M^2} \\
& \left\{ -\frac{u_0 m_s \langle \bar{s}s \rangle f_\phi M_\phi g_\perp^{(v)} (1-u_0)}{3} - \frac{2 \langle \bar{s}s \rangle f_\phi^\perp M_\phi^2 \tilde{C}_\perp (1-u_0)}{3} \right. \\
& + \frac{u_0 m_s \langle \bar{s} g_s \sigma G s \rangle f_\phi M_\phi g_\perp^{(v)} (1-u_0)}{18M^2} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& + \frac{2 \langle \bar{s}s \rangle f_\phi^\perp \phi_\perp (1-u_0)}{3} M^2 E_0(x) - \frac{\langle \bar{s} g_s \sigma G s \rangle f_\phi^\perp \phi_\perp (1-u_0)}{6} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& + \frac{m_Q^4 f_\phi^\perp M_\phi^2 \langle \bar{s} g_s \sigma G s \rangle A_\perp (1-u_0)}{24M^6} + \frac{m_s f_\phi^\perp M_\phi^2 A_\perp (1-u_0)}{8\pi^2} M^2 E_0(x) \\
& - \frac{\langle \bar{s}s \rangle f_\phi^\perp M_\phi^2 A_\perp (1-u_0)}{6} \left( 1 + \frac{m_Q^2}{M^2} \right) + \frac{u_0 \langle \bar{s} g_s \sigma G s \rangle f_\phi^\perp M_\phi^2 \tilde{C}_\perp (1-u_0)}{6M^2} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& \left. - \frac{m_s \langle \bar{s}s \rangle \tilde{f}_\phi M_\phi g_\perp^{(a)} (1-u_0)}{6} \left( 1 + \frac{m_Q^2}{M^2} \right) - \frac{u_0 m_s \langle \bar{s}s \rangle \tilde{f}_\phi M_\phi}{12} \frac{d}{du_0} g_\perp^{(a)} (1-u_0) \right\}, \quad (21)
\end{aligned}$$

$$\begin{aligned}
g_2 = & \frac{1}{\lambda_{\Omega_Q} \lambda_{\Omega_Q^*}} \exp \frac{M_{\Omega_Q}^2 + M_{\Omega_Q^*}^2 - 2u_0(1-u_0)M_\phi^2}{2M^2} \\
& \left\{ \frac{u_0 f_\phi M_\phi [\tilde{\phi}_\parallel (1-u_0) - \tilde{g}_\perp^{(v)} (1-u_0)]}{\pi^2} M^2 E_0(x) \int_0^1 dt t (1-t) e^{-\frac{\tilde{m}_Q^2}{M^2}} \right. \\
& - \frac{u_0 m_Q^2 f_\phi M_\phi [\tilde{\phi}_\parallel (1-u_0) - \tilde{g}_\perp^{(v)} (1-u_0)]}{18M^4} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{u_0 f_\phi M_\phi^3 \tilde{A} (1-u_0)}{4\pi^2} \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} + \frac{u_0 m_Q^2 f_\phi M_\phi^3 \tilde{A} (1-u_0)}{72M^6} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{2u_0 m_s f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp (1-u_0)}{\pi^2} \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{u_0 m_s m_Q^2 f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp (1-u_0)}{9M^6} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{u_0 \tilde{f}_\phi M_\phi}{4\pi^2} M^2 E_0(x) g_\perp^{(a)} (1-u_0) \int_0^1 dt t (1-t) e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{u_0 m_Q^2 \tilde{f}_\phi M_\phi}{72M^4} \langle \frac{\alpha_s GG}{\pi} \rangle g_\perp^{(a)} (1-u_0) \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& \left. + \frac{f_\phi M_\phi}{2\pi^2} M^2 E_0(x) \int_0^1 dt t \int_0^{u_0} d\alpha_{\bar{s}} \int_{u_0-\alpha_{\bar{s}}}^{1-\alpha_{\bar{s}}} d\alpha_g \frac{\mathcal{A}(\alpha_i) + (1-2v)\mathcal{V}(\alpha_i)}{\alpha_g} e^{-\frac{\tilde{m}_Q^2}{M^2}} \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\lambda_{\Omega_Q} \lambda_{\Omega_Q^*}} \exp \frac{M_{\Omega_Q}^2 + M_{\Omega_Q^*}^2 - 2m_Q^2 - 2u_0(1-u_0)M_\phi^2}{2M^2} \\
& \left\{ \frac{2u_0 m_s \langle \bar{s}s \rangle f_\phi M_\phi \left[ \tilde{\phi}_\parallel (1-u_0) - \tilde{g}_\perp^{(v)} (1-u_0) \right]}{3M^2} \right. \\
& - \frac{u_0 m_s \langle \bar{s}s \rangle f_\phi M_\phi^3 \tilde{A} (1-u_0)}{6M^4} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& - \frac{u_0 m_s \langle \bar{s}g_s \sigma Gs \rangle f_\phi M_\phi \left[ \tilde{\phi}_\parallel (1-u_0) - \tilde{g}_\perp^{(v)} (1-u_0) \right]}{9M^4} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& + \frac{8u_0 \langle \bar{s}s \rangle f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp (1-u_0)}{3M^2} - \frac{2u_0 \langle \bar{s}g_s \sigma Gs \rangle f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp (1-u_0)}{3M^4} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& \left. - \frac{u_0 m_s \langle \bar{s}s \rangle \tilde{f}_\phi M_\phi g_\perp^{(a)} (1-u_0)}{6M^2} \right\}, \tag{22}
\end{aligned}$$

$$\begin{aligned}
\text{G3} = & \frac{1}{\lambda_{\Omega_Q} \lambda_{\Omega_Q^*}} \exp \frac{M_{\Omega_Q}^2 + M_{\Omega_Q^*}^2 - 2u_0(1-u_0)M_\phi^2}{2M^2} \\
& \left\{ \frac{f_\phi M_\phi [\tilde{\phi}_\parallel(1-u_0) - \tilde{g}_\perp^{(v)}(1-u_0)]}{2\pi^2} M^4 E_1(x) \int_0^1 dt t(1-t) e^{-\frac{\tilde{m}_Q^2}{M^2}} \right. \\
& - \frac{m_Q^2 f_\phi M_\phi [\tilde{\phi}_\parallel(1-u_0) - \tilde{g}_\perp^{(v)}(1-u_0)]}{36M^2} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1-t}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{f_\phi M_\phi^3 \tilde{A}(1-u_0)}{8\pi^2} M^2 E_0(x) \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{m_Q^2 f_\phi M_\phi^3 \tilde{A}(1-u_0)}{144M^4} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{m_s f_\phi^\perp \phi_\perp(1-u_0)}{2\pi^2} M^4 E_1(x) \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{m_s m_Q^2 f_\phi^\perp \phi_\perp(1-u_0)}{36M^2} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{m_s f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp(1-u_0)}{\pi^2} M^2 E_0(x) \int_0^1 dt t e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{m_s m_Q^2 f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp(1-u_0)}{18M^4} \langle \frac{\alpha_s GG}{\pi} \rangle \int_0^1 dt \frac{1}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& + \frac{\tilde{f}_\phi M_\phi}{8\pi^2} M^4 E_1(x) g_\perp^{(a)}(1-u_0) \int_0^1 dt t(1+t) e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& - \frac{m_Q^2 \tilde{f}_\phi M_\phi}{144M^2} \langle \frac{\alpha_s GG}{\pi} \rangle g_\perp^{(a)}(1-u_0) \int_0^1 dt \frac{1+t}{t^2} e^{-\frac{\tilde{m}_Q^2}{M^2}} \\
& \left. - \frac{u_0 f_\phi M_\phi^3}{2\pi^2} M^2 E_0(x) \int_0^1 dt t \int_0^{u_0} d\alpha_{\bar{s}} \int_{u_0-\alpha_{\bar{s}}}^{1-\alpha_{\bar{s}}} d\alpha_g v \frac{\mathcal{A}(\alpha_i) - \mathcal{V}(\alpha_i)}{\alpha_g} e^{-\frac{\tilde{m}_Q^2}{M^2}} \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\lambda_{\Omega_Q} \lambda_{\Omega_Q^*}} \exp \frac{M_{\Omega_Q}^2 + M_{\Omega_Q^*}^2 - 2m_Q^2 - 2u_0(1-u_0)M_\phi^2}{2M^2} \\
& \left\{ \frac{m_s \langle \bar{s}s \rangle f_\phi M_\phi \left[ \tilde{\phi}_\parallel (1-u_0) - \tilde{g}_\perp^{(v)} (1-u_0) \right]}{3} \right. \\
& - \frac{m_s \langle \bar{s}s \rangle f_\phi M_\phi^3 \tilde{A} (1-u_0)}{12M^2} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle f_\phi M_\phi \left[ \tilde{\phi}_\parallel (1-u_0) - \tilde{g}_\perp^{(v)} (1-u_0) \right]}{18M^2} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& - \frac{2 \langle \bar{s}s \rangle f_\phi^\perp \phi_\perp (1-u_0)}{3} M^2 E_0(x) + \frac{\langle \bar{s}g_s \sigma Gs \rangle f_\phi^\perp \phi_\perp (1-u_0)}{6} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& - \frac{m_Q^4 f_\phi^\perp M_\phi^2 \langle \bar{s}g_s \sigma Gs \rangle A_\perp (1-u_0)}{24M^6} - \frac{m_s f_\phi^\perp M_\phi^2 A_\perp (1-u_0)}{8\pi^2} M^2 E_0(x) \\
& + \frac{\langle \bar{s}s \rangle f_\phi^\perp M_\phi^2 A_\perp (1-u_0)}{6} \left( 1 + \frac{m_Q^2}{M^2} \right) + \frac{4 \langle \bar{s}s \rangle f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp (1-u_0)}{3} \\
& - \frac{\langle \bar{s}g_s \sigma Gs \rangle f_\phi^\perp M_\phi^2 \tilde{\tilde{B}}_\perp (1-u_0)}{3M^2} \left( 1 + \frac{m_Q^2}{M^2} \right) \\
& \left. + \frac{m_s \langle \bar{s}s \rangle \tilde{f}_\phi M_\phi g_\perp^{(a)} (1-u_0)}{12} \left( 1 + \frac{2m_Q^2}{M^2} \right) \right\}, \tag{23}
\end{aligned}$$

where  $M_1^2 = M_2^2 = 2M^2$  and  $u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{1}{2}$  as  $\frac{M_{\Omega_Q^*}^2}{M_{\Omega_Q^*}^2 + M_{\Omega_Q}^2} \approx \frac{1}{2}$ ,  $v = \frac{u_0 - \alpha_{\bar{s}}}{\alpha_g}$ ,  $\tilde{m}_Q^2 = \frac{m_Q^2}{t}$ ,

$E_n(x) = 1 - (1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!})e^{-x}$ ,  $x = \frac{s_0}{M^2}$ , and  $\tilde{f}(1-u_0) = \int_0^{u_0} du \int_0^u dt f(1-t)$ ,  $\tilde{f}(1-u_0) = \int_0^{u_0} du f(1-u)$ , the  $f(u)$  denote the light-cone distribution amplitudes. For some technical details concerning the three particle  $\phi$ -meson light-cone distribution amplitudes, one can consult Ref.[46].

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